Paper Reference(s)

6668/01

Edexcel GCE

Further Pure Mathematics FP2

Advanced Level

Friday 22 June 2012 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics FP2), the paper reference (6668), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 8 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

P40104A

1. Find the set of values of x for which

$$|x^2 - 4| > 3x.$$

(5)

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2. The curve C has polar equation

$$r = 1 + 2\cos\theta, \qquad 0 \leq \theta \leq \frac{\pi}{2}.$$

At the point P on C, the tangent to C is parallel to the initial line.

Given that O is the pole, find the exact length of the line OP.

(7)

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3. (*a*) Express the complex number $-2 + (2\sqrt{3})\text{i}$ in the form $r(\cos\theta + \text{i}\sin\theta)$, $-\pi < \theta \leq \pi$.

(3)

(*b*) Solve the equation

$$z^4 = -2 + (2\sqrt{3})\text{i}$$

giving the roots in the form $r(\cos\theta + \text{i}\sin\theta)$, $-\pi < \theta \leq \pi$.

(5)

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4. Find the general solution of the differential equation

$$\frac{\text{d}^2x}{\text{d}t^2} + 5\frac{\text{d}x}{\text{d}t} + 6x = 2\cos t - \sin t.$$

(9)

5.

$$x\frac{\text{d}y}{\text{d}x} = 3x + y^2.$$

(*a*) Show that

$$x\frac{\text{d}^2y}{\text{d}x^2} + (1 - 2y)\frac{\text{d}y}{\text{d}x} = 3.$$

(2)

Given that $y = 1$ at $x = 1$,

(*b*) find a series solution for y in ascending powers of $(x - 1)$, up to and including the term in $(x - 1)^3$.

(8)

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6. (*a*) Express $\dfrac{1}{r(r+2)}$ in partial fractions.

(2)

(*b*) Hence prove, by the method of differences, that

$$\sum_{r=1}^{n} \frac{1}{r(r+2)} = \frac{n(an+b)}{4(n+1)(n+2)},$$

where a and b are constants to be found.

(6)

(*c*) Hence show that

$$\sum_{r=n+1}^{2n} \frac{1}{r(r+2)} = \frac{n(4n+5)}{4(n+1)(n+2)(2n+1)}.$$

(3)

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7. (*a*) Show that the substitution $y = vx$ transforms the differential equation

$$3xy^2 \frac{\mathrm{d}y}{\mathrm{d}x} = x^3 + y^3 \qquad \text{(I)}$$

into the differential equation

$$3v^2 x \frac{\mathrm{d}v}{\mathrm{d}x} = 1 - 2v^3 \qquad \text{(II)}$$

(3)

(*b*) By solving differential equation (II), find a general solution of differential equation (I) in the form $y = \mathrm{f}(x)$.

(6)

Given that $y = 2$ at $x = 1$,

(*c*) find the value of $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ at $x = 1$.

(2)

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8. The point P represents a complex number z on an Argand diagram such that

$$|z - 6\mathrm{i}| = 2|z - 3|.$$

(*a*) Show that, as z varies, the locus of P is a circle, stating the radius and the coordinates of the centre of this circle.

(6)

The point Q represents a complex number z on an Argand diagram such that

$$\arg(z - 6) = -\frac{3\pi}{4}.$$

(*b*) Sketch, on the same Argand diagram, the locus of P and the locus of Q as z varies.

(4)

(*c*) Find the complex number for which both $|z - 6\mathrm{i}| = 2|z - 3|$ and $\arg(z - 6) = -\dfrac{3\pi}{4}$.

(4)

TOTAL FOR PAPER: 75 MARKS

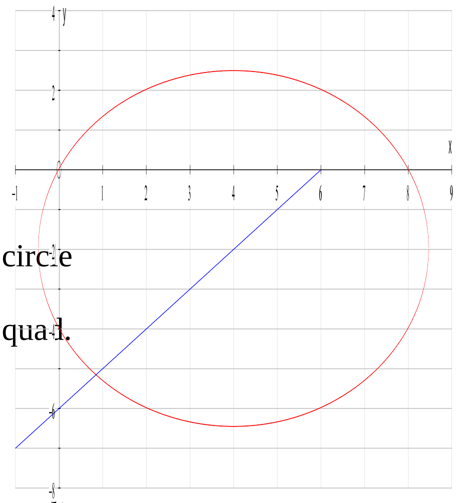
END

Question Number	Scheme	Marks
1.	$x^2-4=3x$ and $x^2-4=-3x$, or graphical method, or squaring both sides, leading to $x=...$	M1
	$(x=-4,\ x=-1)$ $x=1,\ x=4$ **seen anywhere**	B1 B1
	Using only 2 critical values to find an inequality	dM1
	$x<1$ $x>4$ **both strict, ignore 'and'**	A1
		(5)
		5 marks
2.	$y=r\sin\theta=\sin\theta+2\sin\theta\cos\theta$	B1
	$\dfrac{dy}{d\theta}=\cos\theta+2\cos 2\theta$	M1
	$4\cos^2\theta+\cos\theta-2=0$	A1oe
	$\cos\theta=\dfrac{-1\pm\sqrt{1+32}}{8}$	M1 A1
	$OP=r=1+\dfrac{-1+\sqrt{1+32}}{4}=\dfrac{3+\sqrt{33}}{4}$	M1 A1
		(7)
		7 marks
3. (a)	$r=\sqrt{(-2)^2+(2\sqrt{3})^2}=4$	B1
	$\tan\theta=-\sqrt{3}$ (Also allow M mark for $\tan\theta=\sqrt{3}$)	M1
	M mark can be implied by $\theta=\pm\dfrac{2\pi}{3}$ or $\theta=\pm\dfrac{\pi}{3}$	
	$\theta=\dfrac{2\pi}{3}$	A1
		(3)
(b)	Finding the 4th root of their r: $r=4^{1/4}$ $(=\sqrt{2})$	M1
	For one root, dividing their θ by 4: $\theta=\dfrac{2\pi}{3}\div 4=\dfrac{\pi}{6}$	M1
	For another root, add or subtract a multiple of 2π to their θ **and** divide by 4 in correct order.	M1
	$\sqrt{2}(\cos\theta+i\sin\theta)$, where $\theta=-\dfrac{5\pi}{6},\ -\dfrac{\pi}{3},\ \dfrac{\pi}{6},\ \dfrac{2\pi}{3}$	A1 A1
		(5)
		8 marks

Question Number	Scheme	Marks
4.	$m^2+5m+6=0 \qquad m=-2,\ -3$ C.F. $(x=)Ae^{-2t}+Be^{-3t}$ P.I. $x=P\cos t+Q\sin t$ $\dot{x}=-P\sin t+Q\cos t$ $\ddot{x}=-P\cos t-Q\sin t$ $(-P\cos t-Q\sin t)+5(-P\sin t+Q\cos t)+6(P\cos t+Q\sin t)=2\cos t-\sin t$ $-P+5Q+6P=2$ and $-Q-5P+6Q=-1$, and solve for P and Q $P=\frac{3}{10}$ and $Q=\frac{1}{10}$ $x=Ae^{-2t}+Be^{-3t}+\frac{3}{10}\cos t+\frac{1}{10}\sin t$	M1 A1 B1 M1 M1 M1 A1 A1 B1 ft (9) **9 marks**
5. (a)	$x\frac{d^2y}{dx^2}+\frac{dy}{dx}=3+2y\frac{dy}{dx}$ (Using differentiation of product or quotient **and** also differentiation of implicit function) $x\frac{d^2y}{dx^2}+(1-2y)\frac{dy}{dx}=3$ **ag**	M1 A1 cso (2)
(b)	$\left(x\frac{d^3y}{dx^3}+\frac{d^2y}{dx^2}\right)+\ldots\ldots$ $\ldots\ldots\left[(1-2y)\frac{d^2y}{dx^2}-2\left(\frac{dy}{dx}\right)^2\right]=0$ At $x=1$: $\frac{dy}{dx}=4$ $\frac{d^2y}{dx^2}=7 \qquad \frac{d^3y}{dx^3}=32$ $(y=)f(1)+f'(1)(x-1)+\frac{f''(1)(x-1)^2}{2}+\frac{f'''(1)(x-1)^3}{6}\ldots$ $y=1+4(x-1)+\frac{7}{2}(x-1)^2+\frac{16}{3}(x-1)^3$ (or equiv.)	B1 M1 A1 B1 B1, B1 M1 A1 ft (8) **10 marks**

Question Number	Scheme	Marks
6. (a)	$\frac{1}{r(r+2)} = \frac{1}{2}\left(\frac{1}{r} - \frac{1}{r+2}\right) = \frac{1}{2r}, -\frac{1}{2r+4}$	B1,B1oe
		(2)
(b)	$r=1: \quad \frac{1}{2}\left(\frac{1}{1} - \frac{1}{3}\right)$	M1
	$r=2: \quad \frac{1}{2}\left(\frac{1}{2} - \frac{1}{4}\right)$	
	$r=3: \quad \frac{1}{2}\left(\frac{1}{3} - \frac{1}{5}\right)$	
	$r=n-1: \quad \frac{1}{2}\left(\frac{1}{n-1} - \frac{1}{n+1}\right)$	
	$r=n: \quad \frac{1}{2}\left(\frac{1}{n} - \frac{1}{n+2}\right)$	A1
	Summing: $\sum_{r=1}^{n} \frac{1}{r(r+2)} = \frac{1}{2}\left(1 + \frac{1}{2} - \frac{1}{n+1} - \frac{1}{n+2}\right)$	M1 A1
	$= \frac{1}{2}\left(\frac{3(n+1)(n+2) - 2(n+1) - 2(n+2)}{2(n+1)(n+2)}\right) = \frac{n(3n+5)}{4(n+1)(n+2)}$	M1 A1cao
		(6)
(c)	$\sum_{r=1}^{2n} \frac{1}{r(r+2)} = \frac{2n(6n+5)}{4(2n+1)(2n+2)}$	B1oe
	$S_{2n} - S_n \quad = \frac{2n(6n+5)}{4(2n+1)(2n+2)} - \frac{n(3n+5)}{4(n+1)(n+2)}$	M1
	$= \frac{n(6n+5)(n+2) - n(3n+5)(2n+1)}{4(n+1)(n+2)(2n+1)}$	
	$= \frac{n(6n^2+17n+10-6n^2-13n-5)}{4(n+1)(n+2)(2n+1)} = \frac{n(4n+5)}{4(n+1)(n+2)(2n+1)}$ (*ag*)	A1 cso
		(3)
		11 marks

Question Number	Scheme	Marks
7. (a)	$\frac{dy}{dx} = v + x\frac{dv}{dx}$ seen	B1
	$3x^3v^2\left(v + x\frac{dv}{dx}\right) = x^3 + v^3x^3 \quad \Rightarrow \quad 3v^2x\frac{dv}{dx} = 1 - 2v^3$ (**ag**)	M1 A1 cso
		(3)
(b)	$\int \frac{3v^2}{1-2v^3}\,dv = \int \frac{1}{x}\,dx$	M1
	$-\frac{1}{2}\ln(1-2v^3) = \ln x \ \ (+C)$	M1 A1
	$-\ln(1-2v^3) = \ln x^2 + \ln A$	
	$Ax^2 = \frac{1}{1-2v^3}$	M1
	$1 - \frac{2y^3}{x^3} = \frac{1}{Ax^2}$	
	$y = \sqrt[3]{\frac{x^3 - Bx}{2}}$ or equivalent	dM1 A1cso
		(6)
(c)	Using $y = 2$ at $x = 1$: $\quad 12\frac{dy}{dx} = 1 + 8$	M1
	At $x = 1$, $\quad \frac{dy}{dx} = \frac{3}{4}$	A1
		(2)
		11 marks

Question Number	Scheme	Marks
8. (a)	$\lvert x+\mathrm{i}y-6\mathrm{i}\rvert = 2\lvert x+\mathrm{i}y-3\rvert$ $x^2+(y-6)^2=4[(x-3)^2+y^2]$ $x^2+y^2-12y+36=4x^2-24x+36+4y^2$ $3x^2+3y^2-24x+12y=0$ $(x-4)^2+(y+2)^2=20$ Centre $(4,-2)$, Radius $\sqrt{20}=2\sqrt{5}=$ awrt 4.47	M1 M1 A1 M1 A1 A1 (6)
(b)	 Centre in correct quad for their circle Passes through O centre in 4th quad. Half line with positive gradient Correct position, clearly through (6, 0)	M1 A1cao B1 B1 (4)
(c)	Equation of line $y=x-6$ Attempting simultaneous solution of $(x-4)^2+(y+2)^2=20$ and $y=x-6$ $x=4\pm\sqrt{10}$ $\left(4-\sqrt{10}\right)+\mathrm{i}\left(-2-\sqrt{10}\right)$	B1 M1 A1 A1cao (4) **14 marks**